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Cellular automata model of car traffic in a two-dimensional street network

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Abstract. We propose a road traffic cellular automata model suitable for an urban environment. North, east, south and west car displacements are possible and road crossings are naturally implemented as rotary junctions. We consider the traffic in a Manhattan-like city and study the flow diagram and the car density profile along road segments. We observe that the length of the car queues obeys a complex dynamics and is not uniform across the network. The street length between two junctions and the turning strategies at rotaries are relevant parameters of the model. Our results are also confirmed by fully continuous traffic simulations.

1. Introduction

Cellular automata models for road traffic have received a great deal of interest during the past few years (see [1–5] for instance).

One-dimensional models for single-lane car motions are quite simple and elegant. The road is represented as a line of cells, each of them being occupied by a vehicle or not. All cars travel in the same direction (say to the right). Their positions are updated synchronously, in successive iterations (discrete time steps). During the motion, each car can be at rest or jump to the nearest-neighbour site, along the direction of motion. The rule is simply that a car moves only if its destination cell is empty. This means that the drivers are short-sighted and do not know whether the car in front will move or whether it is also blocked by another car. Therefore the state of each cell s_i is entirely determined by the occupancy of the cell itself and its two nearest neighbours s_{i-1} and s_{i+1}. The motion rule can be summarized by the following table, where all eight possible configurations $(s_{i-1}s_is_{i+1})_t \rightarrow (s_i)_{t+1}$ are given:

$$\underbrace{(111)}_{1} \quad \underbrace{(110)}_{0} \quad \underbrace{(101)}_{1} \quad \underbrace{(100)}_{1} \quad \underbrace{(011)}_{1} \quad \underbrace{(010)}_{0} \quad \underbrace{(001)}_{0} \quad \underbrace{(000)}_{0} . \qquad (1)$$

This cellular automaton rule turns out to be Wolfram's rule 184 [6, 1].

This simple dynamics captures an interesting feature of real car motion: traffic congestion. Suppose we have a low car density ρ in the system, for instance something like

$$\ldots 0010000010010000010 \ldots . \qquad (2)$$

This is a *free* traffic regime in which all the cars are able to move. The average velocity $\langle v \rangle$ defined as the number of motions divided by the number of cars is then

$$\langle v_f \rangle = 1 \qquad (3)$$

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where the subscript f indicates a free state. On the other hand, in a high-density configuration such as

$$\ldots 110101110101101110 \ldots \tag{4}$$

only 6 cars out of 12 will move and $\langle v \rangle = \frac{1}{2}$. This is a partially jammed regime.

If the car positions were uncorrelated, the number of moving cars (i.e. the number of particle–hole pairs) would be given by $L\rho(1-\rho)$, where L is the system size. Since the number of cars is ρL, the average velocity would be

$$\langle v_{\mathrm{uncorrel}} \rangle = 1 - \rho \, . \tag{5}$$

However, in this model, the car occupancy of adjacent sites is highly correlated and the vehicles cannot move until a hole has appeared in front of them. The car distribution tries to self-adjust to a situation where there is one space between consecutive cars. For density less than half, this is easily realized and the system organizes to have one car every other site.

Therefore, due to these correlations, equation (5) is wrong in the high-density regime. In this case, since a car needs a hole to move to, we expect that the number of moving cars simply equals the number of empty cells [1]. Thus, the number of motions is $L(1-\rho)$ and the average velocity in the jammed phase is

$$\langle v_j \rangle = \frac{1-\rho}{\rho} \, . \tag{6}$$

Yukawa and co-workers [1] have studied an interesting extension of the above model. They consider a periodic system, with a special site which blocks incoming cars and releases them with probability r, provided that the next site is free. Depending on the car density, three different regimes are observed, namely (i) a free phase, (ii) a constant flow phase and (iii) a jammed phase. A simple mean-field description of this system is possible and the critical densities are found to be a function of r.

Here we would like to extend this problem to the situation of a street network. This requires us to define new cellular automata rules in order to deal with several cars entering the same road junction. In addition, a driver's decision at crossing (going straight or turning) should be defined. As explained in section 2, our approach is to model a road intersection as a rotary. Cars in the rotary have priority over those wishing to enter. Various strategies can be considered for cars in a rotary to determine their behaviour. For instance, biased decisions are easily implemented in order to mimic traffic lights. It is also possible to add in the model a trip plan (i.e. some final destination to each vehicle). As a result, more complex problems [7, 8] could be investigated in the framework of our approach.

In this model, road crossings are a bottleneck limiting traffic flow. However, as opposed to [1], this bottleneck is not controlled by an external parameter r. The system responds dynamically to the demand and selects its own flow.

In section 3, we will consider a mean-field description of our model and study how the distance between two consecutive junctions affects the flow diagram.

As described in section 4, different turning strategies at rotaries are found to play a role. Some of them are likely to produce junction deadlocks, a situation in which no more cars can ever move at a crossing. The jammed region grows as more cars arrive and propagates throughout the entire system. After a while, all cars get stuck and this results in a first-order transition to a fully jammed state.

Finally, in section 5 we will also provide a detailed study of the car density profile along streets.

Our approach differs from other two-dimensional cellular automata traffic models [9–11] in two respects. First, these models do not have the concept of streets. Cars can be anywhere in a two-dimensional lattice and can typically move to any nearest-neighbour empty site.

Second, they have a rather primitive motion rule in order to avoid double occupancy. They consider an alternate update of each directions of motion, as if there were synchronized traffic lights at each lattice site, allowing horizontal motion at odd time steps and vertical motion at even time steps.

In that sense, our approach is more realistic for describing traffic in a city. However, in the limit where the distance between the rotaries becomes small (four lattice sites is the smallest valid separation in our model), we obtain a flow pattern similar to those observed in [9, 10].

2. The street network model

According to the basic ideas discussed in section 1, a cellular automaton rule of car motion can be expressed by the following relation:

$$n_i(t+1) = n_i^{\text{in}}(t)(1-n_i(t)) + n_i(t)n_i^{\text{out}}(t) \tag{7}$$

where $n_i(t)$ is the car occupation number ($n_i = 0$ means a free site, $n_i = 1$ means that a vehicle is present at site i). The quantity $n_i^{\text{in}}(t)$ represents the state of the source cell, i.e that from which a car may move to cell i. Similarly, $n_i^{\text{out}}(t)$ indicates the state of the destination cell, i.e that the car at site i would like to move to. Rule (7) means that the next state of cell i is 1 if a car is currently present and the next cell is occupied, or if no car is currently present and a car is arriving.

Equation (7) can also be written as a balance equation. One has

$$n_i(t+1) - n_i(t) = n_i^{\text{in}}(1-n_i) - n_i(1-n_i^{\text{out}}) \tag{8}$$

where the right-hand side is computed at time t. The quantity

$$j_i^{\text{out}} = n_i(1-n_i^{\text{out}}) \tag{9}$$

is the number of cars (0 or 1) leaving cell i between iteration t and $t+1$ (i.e those which find n^{out} empty). Similarly,

$$j_i^{\text{in}} = n_i^{\text{in}}(1-n_i) \tag{10}$$

is the number of cars (0 or 1) entering cell i between iteration t and $t+1$. Taking an ensemble average, equation (8) reads

$$\rho_i(t+1) - \rho_i(t) = J_i^{\text{in}}(t) - J_i^{\text{out}}(t) \tag{11}$$

where $\rho_i(t) = \langle n_i(t)\rangle$ is the car density at site i and time t and $J = \langle j \rangle$ is the average flow. This is the continuity (or car conservation) equation of our dynamics.

In a closed system, the total number of vehicles $N^{mvt}(t)$ that have moved between two consecutive iterations is

$$N^{mvt}(t) = \sum_{i=1}^{L} j_i^{\text{out}}(t) = \sum_{i=1}^{L} j_i^{\text{in}}(t) \tag{12}$$

where L is the system size (number of cells). Thus, in a homogeneous, stationary system, the average car flow $J = J_i^{\text{out}} = J_i^{\text{in}}$ can be obtained as

$$J = \frac{N^{mvt}}{L} = \frac{N^{mvt}}{N_{\text{car}}}\frac{N_{\text{car}}}{L} = \rho\langle v\rangle \tag{13}$$

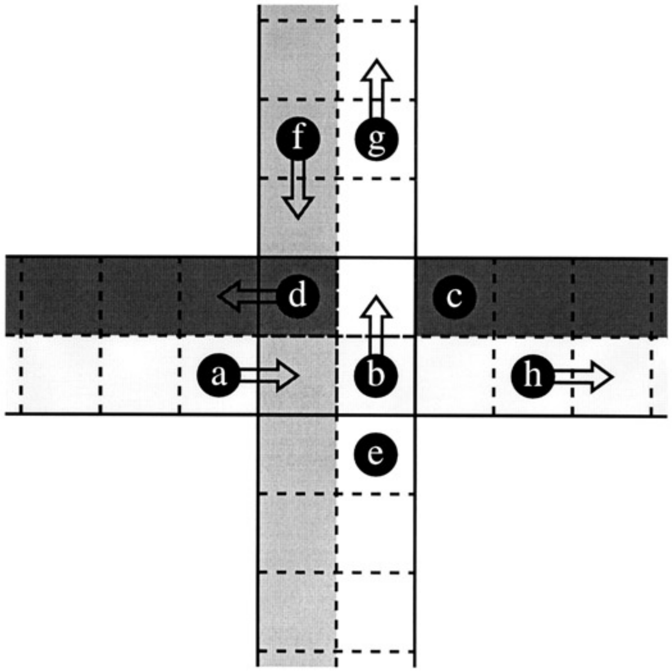


Figure 1. Example of a traffic configuration near a junction. The four central cells represent a rotary which is travelled counterclockwise. The gray levels indicate the different traffic lanes: white is a northbound lane, light gray an eastbound lane, gray a southbound lane and, finally, dark gray is a westbound lane. The dots labelled a, b, c, d, e, f, g and h are cars which will move to the destination cell indicated by the arrows, as determined by the cell turn flag F. Cars without an arrow are forbidden to move.

where $\langle v\rangle$ is the average velocity defined in section 1. Note that J can also be interpreted as the probability of a car motion.

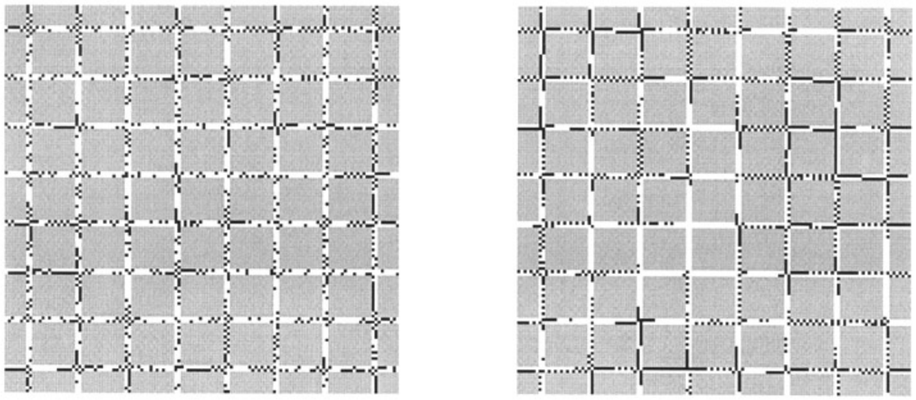
In order to discuss further rule (7), we have to specify how n_i^{in} and n_i^{out} are defined in terms of occupation numbers. For a simple one-dimensional periodic road, their expression is very simple

$$n_i^{\text{in}} = n_{i-1} \qquad n_i^{\text{out}} = n_{i+1} \tag{14}$$

and the microdynamics reduces exactly to rule 184 of Wolfram.

For a two-dimensional road network, these quantities can be generalized as follows. We assume that horizontal roads consist of two lanes, one for eastward motion and the other for westward motion. Similarly, vertical streets are composed of northbound and southbound lanes. The question is to define the motion rule of the cars at a road junction. This can be realized in a simple way if one assumes that a rotary is located at each crossing. Thus, road junctions are formed by central points around which the traffic always moves in the same direction. A vehicle in a rotary has the priority over any car entering.

The implementation we propose for this rule is illustrated in figure 1, where a four-corner junction is shown. The four middle cells constitute the rotary. A vehicle on the rotary (like b or d) can either rotate counterclockwise or exit. A local flag F is used to decide the motion of a car in a rotary. If $F = 0$, the vehicle (like d) exits in the direction allowed by the colour of its lane (see the figure caption). If $F = 1$, the vehicle moves counterclockwise, like b. The value of the local turn flag F can be updated according to the



(*a*) (*b*)

Figure 2. Traffic configuration after 600 iterations, for a car density of 30%. Streets are white, buildings gray and the black pixels represent the cars. Situation (*a*) corresponds to an equally likely behaviour at each rotary junction, whereas (*b*) mimics the presence of traffic lights. In the second case, queues are more likely to form and the global mobility is less than in the first case.

modelling needs: it can be constant for some amount of time to impose a particular motion at a given junction, completely random, random with some bias to favour a direction of motion, or may change deterministically according to any user-specified rule.

As in the one-dimensional rule, a vehicle moves only when its destination cell n^{out} is empty. Far from a rotary, the state of the destination cell is determined by the occupation of the down-motion cell. This is also the case for a vehicle turning in the rotary. On the other hand, a car wanting to enter the rotary has to check two cells because it does not have priority. This check is made by looking at the turn flag F of the neighbouring cells with priority.

For instance, car c cannot enter the rotary because b is going to move to the white cell. The car e cannot move either because it sees b (and cannot know whether or not b will actually move). Car a, on the other hand, can enter because it sees that d is leaving the rotary and that the gray cell ahead is free.

Similarly, the incoming vehicle n^{in} to a given cell is computed differently inside and outside of the rotary. The light gray cell occupied by car b has two possible inputs: with priority, it is the vehicle from the gray cell to the west; if this cell is empty, the input will be the incoming lane, namely the car labelled e.

Figure 2 shows typical traffic configurations in a Manhattan-like city. In figure 2(a), a vehicle has a probability $\frac{1}{2}$ to exit at each rotary cell. In figure 2(b), the turn flag F has an initial random distribution on the rotary. This distribution is fixed for the first 20 iterations and then flips to $F = 1 - F$ for the next 20 steps and so on. In this way, a junction acts as a traffic light, which for some amount of time allows only a given flow pattern (note that right turns on red is permitted). We observed that the global traffic pattern is different in the two cases: in (a), the car distribution is quite homogeneous along the streets. On the other hand, in (b), cars queue at some junctions while some other streets remain empty.

3. The mean-field solution

In this section, we derive a mean-field description of the traffic flow, in a steady-state situation. We extend the derivation of [1] to our problem. The first quantities of interest are the *average* car density ρ and the average speed $\langle v\rangle$ defined in section 1.

The first ingredient in the mean-field analysis is the fact that a rotary junction has a maximum possible flow of cars. That is, the number of vehicles able to enter a rotary per unit time cannot be larger than a given value determined by the rule of motion. Thus there is a critical average density $\rho_1^{\rm crit}$ above which the traffic is not free but constrained by this maximum rotary flow. As a result, car queues are formed at road junctions.

The second key observation is that, in the regime above $\rho_1^{\rm crit}$, the system self-organizes in three different regions of fixed car densities: the queues that form before a junction, the road segments after a junction, characterized by a low traffic density and the region inside a rotary. The three densities associated with these different regions correspond to a jammed density ρ_j, a free traffic density ρ_f and a rotary density ρ_r, respectively.

As the overall car number is increased, ρ_j, ρ_f and ρ_r remain constant: the result of increasing the number of cars is to extend the length ℓ of the car queues, without changing the density in the three regions. The reason for fixed densities is that, due to the flow diagram of rule 184 [1], there are only two possible densities ρ_f and ρ_j compatible with a given traffic flow $\rho\langle v\rangle$, along a road segment. Thus, the only way to absorb an excess of cars is to increase the size of the queue.

When one keeps adding cars in the system, there is a second critical average density $\rho_2^{\rm crit}$ for which the length of some queues becomes larger than the distance separating two consecutive street intersections. The up-traffic rotary output gets disturbed and, from a maximum-flow traffic regime, one gets into a strongly jammed phase.

The values of ρ_f, ρ_j and ρ_r can be obtained by simple mean-field like arguments, assuming that all queues are of the same length. From equation (11), we find that a stationary state is characterized by a constant traffic flow. We will apply this condition in the three regions discussed earlier and at the interface between these regions: the traffic flow $J_{\rm queue}$ in the queues equals the entering flow $J_{\rm rotary}^{\rm in}$ of a certain direction in the rotaries which, in turn, equals the flow $J_{\rm rotary}^{\rm out}$ out of the rotaries and the flow $J_{\rm free}$ in the street just after the junctions:

$$J_{\rm queue} = J_{\rm rotary}^{\rm in} = J_{\rm rotary}^{\rm out} = J_{\rm free}\,. \tag{15}$$

From equations (13), (6) and (3), we obtain

$$J_{\rm queue} = \rho_j\langle v_j\rangle = 1-\rho_j \qquad \text{and} \qquad J_{\rm free} = \rho_f\langle v_f\rangle = \rho_f \tag{16}$$

from which we conclude that

$$\rho_f = 1-\rho_j\,. \tag{17}$$

The expressions for $J_{\rm rotary}^{\rm in}$ and $J_{\rm rotary}^{\rm out}$ (which are the probabilities of entering and exiting the intersection, respectively) depend on the rule of motion in a rotary. In the implementation we have discussed in section 2, the motion of several cars in a rotary is strongly correlated. Since two cars cannot occupy simultaneously two consecutive cells of the rotary and move, the stationary solution is, on average, two cars per rotary, in a diagonal configuration. Thus,

$$\rho_r = \tfrac{1}{2}\,. \tag{18}$$

Since a car keeps turning in a rotary with probability $\frac{1}{2}$, the probability of exiting a rotary is

$$J_{\rm rotary}^{\rm out} = \tfrac{1}{2}\rho_r\,. \tag{19}$$

Note that the probability $1-\rho_f$ of having a free cell just outside the rotary does not multiply the right-hand side of this formula because the two events are not independent. Indeed, due to the alternation of vehicle and hole in a rotary, the exit of a rotary is always free in the maximum flow regime.

Likewise, entering a rotary (say from the left) is possible when the lower left cell of the rotary is empty and the vehicle in the upper left cell (if any) wants to exit the rotary westward. Thus, we have

$$J_{\text{rotary}}^{\text{in}} = \tfrac{1}{2}(1-\rho_r)\,. \tag{20}$$

As before, the occupation probability ρ_j of the cell just before the junction does not appear because, due to the correlations, at the time the entry is possible, there is always a car waiting for this.

From equations (15) and (17), we then conclude that

$$\rho_f = \tfrac{1}{4} \qquad \rho_j = \tfrac{3}{4} \qquad \rho_r = \tfrac{1}{2}\,. \tag{21}$$

In this mean-field description, the maximum flow regime lasts until the tail of the car queue reaches the up-traffic junction. Assuming that the queues are of the same length ℓ along all road segments and that the separation between two consecutive junctions is L (the network period), we can relate [1] the average car density ρ to ℓ by the relation

$$4(L-2-\ell)\rho_f + 4\ell\rho_j + 4\rho_r = 4L\rho\,. \tag{22}$$

Equation (22) simply reflects the fact that the total number of cars is distributed in three regions: queues of length ℓ and density ρ_j, free traffic segments of length $L-\ell-2$ and density ρ_f and rotaries of size four and density ρ_r. For this calculation, we have considered a basic network element, namely one rotary with two entering and two exiting lanes.

Thus ℓ is given by

$$\frac{\ell}{L} = \frac{\rho-\rho_f}{\rho_j-\rho_f} + \frac{2\rho_f-\rho_r}{(\rho_j-\rho_f)L}\,. \tag{23}$$

In the case of large L, we can approximate this result as

$$\frac{\ell}{L} = \frac{\rho-\rho_f}{\rho_j-\rho_f}\,. \tag{24}$$

Equation (24) provides a way to determine the critical densities ρ_1^{crit} and ρ_2^{crit}. For $\rho < \rho_f$, ℓ is negative, which should be interpreted in the sense that no queue is formed. This is the free traffic regime. Thus, $\rho_1^{\text{crit}} = \rho_f = \frac{1}{4}$ and the average velocity is $\langle v\rangle = 1$, independent of ρ.

On the other hand, for $\rho_f < \rho < \rho_j$, car queues form but their lengths are smaller than the distance between successive intersections. This is the maximum flow regime. In this case, we have $\rho\langle v\rangle = J = \text{constant} = \frac{1}{4}$, that is $\langle v\rangle = 1/(4\rho)$.

Finally, for $\rho > \rho_j = \rho_2^{\text{crit}}$, the queues reach their maximum length L and the rotary exits are hindered. This is the strongly jammed traffic regime. The traffic velocity is governed by the motion of holes and obeys (6), namely $\langle v\rangle = (1-\rho)/\rho$.

If $\langle v\rangle$ is taken as the order parameter, both of these transitions are second order. Actual simulations of our cellular automata street network are well described by the above mean-field description, provided the turning decision at rotaries is random and not time-correlated. This means that a driver will not stick to a decision: if a rotary exit is blocked at time t, he might very well decide to keep turning in the rotary at time $t+1$ (figure 3(*a*) shows the velocity-density diagram, for this case).

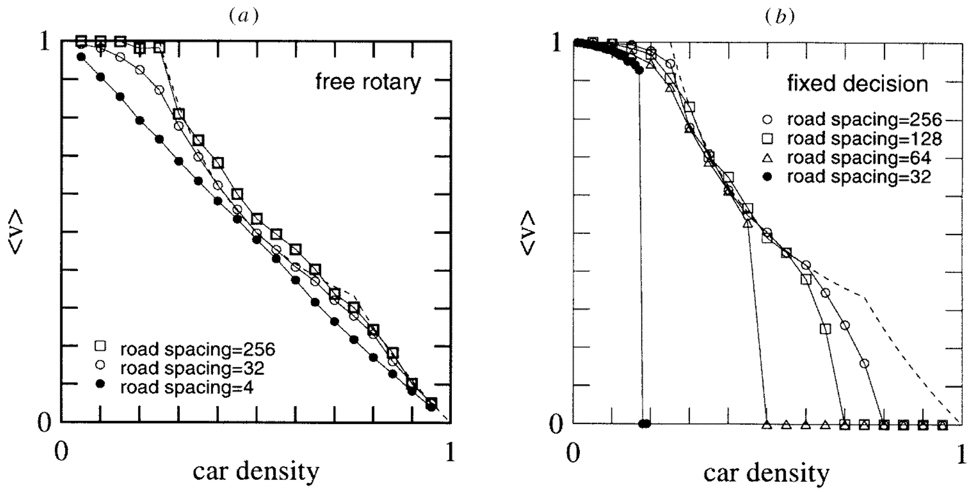


Figure 3. Average velocity versus average density for the cellular automata street network, for (a) time-uncorrelated turning strategies and (b) a fixed driver's decision. The different curves correspond to different distances L between successive road junctions. The broken curve is the mean-field prediction. Junction deadlock is likely to occur in (b), resulting in a completely jammed state.

We have considered various road spacings for our measurements (i.e the distance L separating consecutive intersections). The larger the spacing the better the agreement with the mean-field description. Clearly, finite-size effects play an important role and are certainly not negligible in urban traffic since the road segments cannot be considered as infinite.

However, as the junction spacing decreases, we observe in figure 3(a) a different behaviour: whereas the features of a network of streets, in the limit of large L, is quite similar to one single lane with controlled flow (see [1]), the case of very small L ($L = 4$ is the smallest distance allowed by the model) yields a linear dependence of $\langle v \rangle$ upon ρ. This behaviour can be understood as follows: the car displacement along a line results in a very correlated motion, as discussed in section 1. In the case of short road segments, the locations of cars are little correlated because they rarely follow each other. Therefore, the number of cars moving at time t can be approximated by (5). The behaviour of our model for small L has similarities with the one obtained by Cuesta *et al* in [10], which is expected since $L \to 1$ would correspond to so-called two-dimensional traffic. However, in the situation described here, we do not observe a first-order phase transition, as in [10]. We shall return to this question in the next section.

4. Junction deadlocks

The results presented in the previous section assume that drivers are free to change their mind if an intersection is momentarily locked. As a result, the load on each road segment is well balanced and, as long as there is a hole in the network, motion will occur.

A more realistic strategy at a road junction is to stick to the first decision (which is still random) and stop until the destination cell is freed. This modification of the car behaviour at a rotary has a dramatic consequence in our model (see figure 3(b)). Suppose that four cars simultaneously enter an empty rotary and all decide to continue in this rotary for their next move. But, since there is no hole in the rotary, no motion is possible. If driver's decisions are frozen, there is a deadlock at the intersection, from which one cannot recover. Further incoming cars will queue up and gradually decrease the network capacity until complete jamming is reached. Therefore, there is an abrupt jump in the velocity diagram, from $\langle v\rangle = \langle v(\rho)\rangle$ to $\langle v\rangle = 0$. This can be interpreted as a first-order phase transition, from partial to complete jamming.

However, although the probability of such a junction deadlock increases with the car densities, it is non-zero even for small ρ. Therefore, the flowing regime in which cars can move is not stable: waiting long enough will eventually yield such a blocking event. But as long as this event does not take place, a well defined relation between $\langle v\rangle$ and $\langle \rho\rangle$ holds. In this sense, we may say that the flowing regime is *metastable*.

For a given observation time (we have typically considered 2000 simulation steps), complete jamming almost certainly occurs if the density is above some critical value. As the road segments between intersections act as car reservoirs, absorbing a local traffic excess, this critical value is higher as L grows.

There is, however, a way to reduce the risk of a junction deadlock considerably. We can force the vehicles in a rotary to keep moving and turning until the exit they want is free. Thus, four cars at the same junction do not block it. This dynamics is yet not completely deadlock free: one can exhibit configurations in which the four rotaries around the same building are mutually locked (all cars willing to go to the jammed exit). As in real traffic, such a situation cannot evolve unless a driver accepts a change of his destination. In our numerical simulations, however, complete jamming has almost never been observed in a finite time interval, with this new strategy.

According to this new behaviour, we modify the rule of motion in a rotary: either a car exits or it turns around. Consequently, two cars can follow each other in a rotary without leaving a hole between them. As a result, the occupation of the cell right before and right after the rotary is no longer correlated with the location of cars in the rotary. Equation (15)

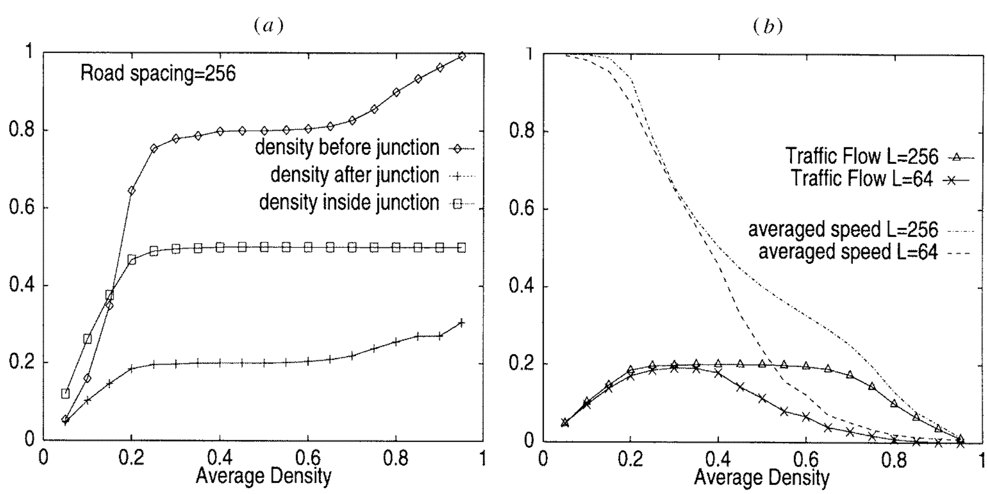


Figure 4. Fixed decision model, with modified rotary motion preventing deadlocks. (*a*) Car density in the three traffic regions (free regions, queues and rotaries), as a function of the average density, for a road spacing $L = 256$. (*b*) Traffic flow and average velocity for two different road spacings.

has to be rewritten with

$$J^{\mathrm{in}}_{\mathrm{rotary}} = \rho_j \frac{1-\rho_r}{2} \qquad J^{\mathrm{out}}_{\mathrm{rotary}} = (1-\rho_f)\frac{\rho_r}{2} \tag{25}$$

and its solution is

$$\rho_j = \tfrac{4}{5} \qquad \rho_r = \tfrac{1}{2} \qquad \rho_f = \tfrac{1}{5}\,. \tag{26}$$

Thus, the strategy of fixed decision obeys the same mean-field description as discussed in section 3. The critical values ρ_1^{crit} and ρ_2^{crit} are just renormalized to the new values of ρ_f and ρ_j given in equation (26).

These mean-field predictions have been checked numerically in figure 4. As expected, we observe three regimes: free traffic, maximum flow and strongly jammed. The vehicles are distributed in the three regions discussed previously (before, after and inside a junction), each with a specific density. In addition, the numerical simulations show good agreement with the mean-field location of the first transition (from the free regime to the maximum flow regime), at $\rho_1^{\mathrm{crit}} = \rho_f = 0.2$. This is, however, not the case of the second transition at $\rho_2^{\mathrm{crit}} = \rho_j = 0.8$, which clearly depends on the road spacing L.

5. Queue density profile

The reason why the second transition is not well described by the mean-field approximation is that the value ℓ given by equation (24) fluctuates considerably and some queues grow at the expense of the others. The traffic is not distributed uniformly and some road segments are much more loaded than the others. The load distribution changes with time, as a result of the 'microscopic' fluctuations. A complex dynamics of car queues length is observed over the network, as illustrated in figure 5.

Because of these fluctuations, some road junctions get disturbed by a down-traffic queue growing up to size L, even for average densities $\rho < \rho_j$. As a result, the maximum flow

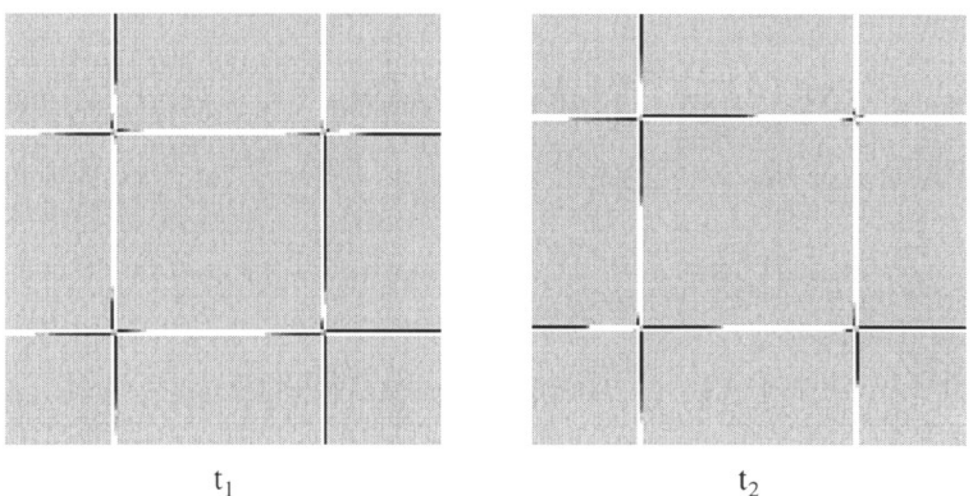


Figure 5. Fluctuation of car queue length, for the rule with fixed turning decision. Buildings are the large gray square blocks and streets are shown in white. Car queues are indicated by the black lines. The gray tails of the queues are due to density variation during the time average. The same network portion is represented on the left and on the right, at two different times t_1 and $t_2 > t_1$. The traffic load distribution has changed from one configuration to the other.

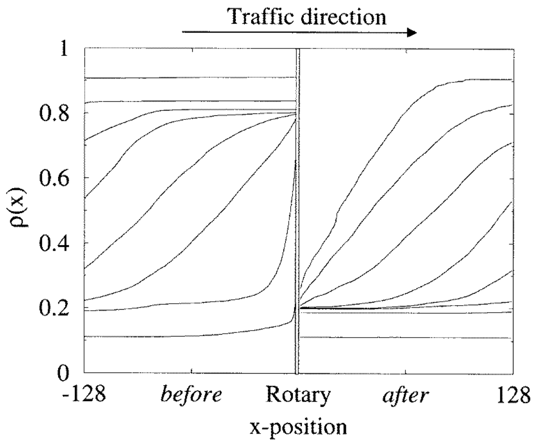


Figure 6. Time and space average of the car density profile along the segments of the street network. The road spacing is $L = 256$ and the system is periodic with one rotary located at $x = 0$. By space average, we mean that we have summed the contributions of the four traffic lanes (north, south, east and west). Each of the eight curves correspond to a different average density, namely $\rho = 0.1, 0.2, \ldots, 0.8$.

regime ends much earlier than expected, i.e. for a critical density smaller then ρ_2^{crit}. This effect is amplified as the rotaries get closer to each other because the road segments are not long enough to absorb the fluctuations. Equation (24) predicts a ratio ℓ/L independent of L. Our observation is that this is clearly wrong when each road segment is considered separately (the $1/L$ correction in equation (23) is too small to explain the importance of the effect). However, the cumulative length of all queues seems to obey equation (24) well [12].

In order to provide a more quantitative description of these fluctuations, we have measured the average density profile between two consecutive junctions. Over a long period of time and for each location along the road segments, we have observed the average cell occupation. Since the density is ρ_j in a queue and ρ_f outside, our measurement indicates the portion of time a given site belongs to the queue. These results are summarized in figure 6. The dependence of ρ_2^{crit} upon ρ and L is still under investigation.

The situation described in figure 6 has only one intersection (crossing of two roads). In the case of several rotaries, we obtain the same qualitative behaviour except that some symmetry breaking is observed among the junctions. There is a checkerboard pattern, which makes the 'white' rotaries have a different density profile than the 'black' ones. However, a more detailed analysis is required before we can confirm the generality of this effect.

6. Conclusions

A first conclusion is that for a road spacing L in the range of what is expected for an urban area, the behaviour of our model has more similarities with the one-dimensional traffic model of [1] than with the two-dimensional cellular automata models of [9, 10]. In that sense, we may say that a street network is not a fully two-dimensional dynamics. The length of the road segments between the junctions can absorb local traffic excess. Queues are formed and correlations build up. Our rotaries also act as a flow capacity limitation but, in contrast with [1], this limitation results from the rule of motion and is not enforced externally. Fixed capacity devices result in three dynamical regimes (free traffic, maximum flow and strongly jammed).

However, the behaviour of a street network exhibits new features as compared with single-lane models: metastability of the flow diagram and junction deadlocks (also called gridlocks). The resulting first-order phase transition is similar to what is observed in 2D traffic models.

In addition, the quantity L has a direct impact on the behaviour of the system, for large densities (for low densities it causes finite-size effects). Also, the overall dynamics is quite sensitive to the driver's behaviour at rotaries for choosing his destination. Note that our model can be extended so as to give a different trip plan to each vehicle.

Another observation is that, although the road network is homogeneous and all drivers similar, the traffic load is not distributed uniformly in space and time. Some regions get more congested than others and some junctions get clogged, thus reducing the overall traffic flow more than expected from the mean-field calculations.

The structure in queues that play an important role in our model suggests that a more macroscopic level of description of traffic in urban areas could be envisaged directly with the quantities ℓ_{ij}, defined as the length of the queues between junction i and j. This may lead to more effective numerical simulations [7].

Recent CA traffic models [3, 8] consider more sophisticated rules. Multispeed car motion is found to be a crucial ingredient to describe highway traffic and phenomena such as start–stop waves. However, for urban traffic, these features are less important and interactions at road junctions are certainly the dominant effect.

This observation is confirmed by other microtraffic simulations we have investigated. In addition to the CA model, we have developed [12] an off-lattice continuous traffic model in which each vehicle can have any velocity, position and acceleration within a given range. This simulator has been implemented on a Connection Machine CM-200 and proceeds along the same lines as the simulator PARAMICS [13], except for the implementation details.

In a situation similar to that studied with our CA model, we observe the same emergent behaviour as described in this paper. Of course, the continuous microtraffic simulator provides more flexibility to study fine aspects, such as the role of the speed limit. However, the macroscopic features of the CA model are well reproduced.

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